Bulletin of the Seismological Society of America. Vol. 65, No. 1, pp. 133–138. February 1975

A SIMPLIFICATION IN THE CALCULATION OF MOTIONS NEAR A PROPAGATING DISLOCATION

BY JOHN BOATWRIGHT AND DAVID M. BOORE

ABSTRACT

From Haskell's (1969) integral representations for the near-field displacements due to a propagating strike-slip and dip-slip dislocation, a solution is obtained for a dislocation "line source" by an analytic integration in the direction of the fault propagation. This reduces the numerical integration from a surface integral required for the usual evaluation of the near-field motion, to a one-dimensional integration over the fault width. Since the dislocation function modeled here is a Heaviside step function, these results may be extended to any arbitrary source time-function by convolving these displacements with the time derivative of the desired source function.

INTRODUCTION

The ability to synthesize seismic motions produced by propagating faults of finite extent is of basic importance in the rapidly developing field of near-field seismology. One of the most common methods in current use is based on Haskell (1969). In this method a Green's function is integrated numerically over the fault surface. In principle, this method can simulate arbitrarily complex fault propagation, with the source time-function and rupture velocity given as spatial functions over a fault surface of any shape. In practice, however, most simulations (e.g., Anderson, 1974; Trifunac, 1974; Tsai and Patton, 1972; Battis and Turnbull, 1974) simplify the description of the fault considerably and treat a rectangular fault which ruptures instantaneously over the fault width and propagates at a uniform speed until the fault terminates. A more complex model of faulting can be obtained by summing the contributions from a number of these simple rectangular faults.

It is possible to evaluate the motion from a simple rectangular fault model by first breaking the rectangle into a number of strips of small width. An analytic expression is given in this paper for the contribution due to the propagation along each strip. Thus the usual two-dimensional numerical integration is reduced to a simple sum over the contributions for each strip. Although the evaluation of the motion for a propagating strip was first solved by Knopoff and Gilbert (1959), an error was pointed out by Savage (1965) which casts doubt on the expressions contained in Knopoff and Gilbert (1959). Savage rederived the expressions and published wave forms based on the resulting equations (Savage, 1965), but did not publish the expressions themselves. Unfortunately, his derivations have since been lost (Savage, personal communication, 1973). With his encouragement we present here the solutions with the hope that other seismologists will find them useful in simulating near-field wave motions.

THE MODEL

We have assumed for a dislocation source function a propagating Heaviside step function. Thus the discontinuity across the fault surface (Δu_k) is given by

$$\Delta u_k = D_k H(t-\xi_1/v)[H(\xi_1)-H(\xi_1-L)] \tag{1}$$

where v is the rupture velocity and L is the fault length. D_1, D_2 correspond to dislocation slip in the positive x_1, x_2 directions, respectively, on the positive x_3 side of the fault. (See Figure 1 for the geometry.)

The integration of Haskell's operator $M_{ij,q}$ across the fault "strip", [see Haskell, 1969, equations (2), (3.1–4.3)], was simplified through a transformation of variables from ξ_1, the first component of the fault coordinates, to r_1, the first component of the vector describing the relative position of the observation point to a point on the fault surface. The resulting displacements were artificially decoupled into P and S modes, i.e.,

$$u_i = u_i^P + u_i^s$$

where u_i is the ith component of displacement at the observation point. These "decoupled" wave motions are valid by themselves only for the study of first motions, since coupling terms (which represent a spectrum of velocities from α to β), are included in $u^P{}_i$ and $u^s{}_i$. The sum of the P and S modes, however, correctly describes the total motion for all times.

The integrated solutions (as functions of the variable of integration, ζ) are listed for

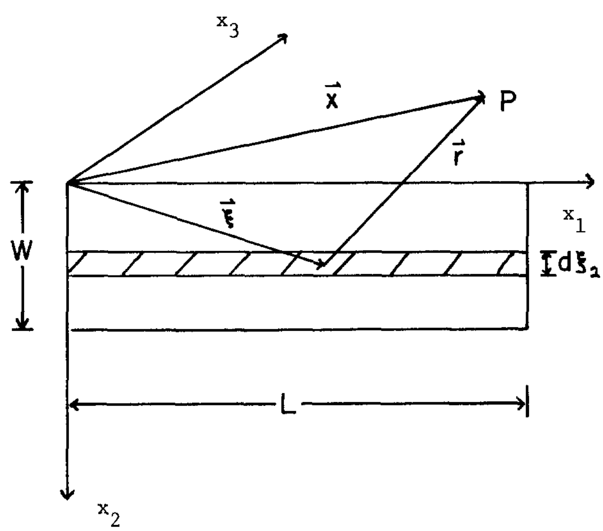

FIG. 1. The fault of width W is broken into a number of strips of width $d\xi_2$, located at ξ_2. The motion on each strip starts at $x_1 = 0$ and ends at $x_1 = L$. $P(x_1, x_2, x_3)$ is the observation point.

both strike-slip and dip-slip dislocations, (D_1 and D_2) and are divided in a natural way into three sets of terms; the far-field terms (I_{ik}^m), the coupling terms (H_{ik}), and the terms which give rise to finite deformations (F_{ik}^m).

These results have been thoroughly checked, in unconvolved form against Savage's (1965) results, and in a convolved form, with a numerical integration over the width, against some results supplied by John Anderson (written communication, 1974). The reader should be warned that some of the figures in Haskell's (1969) paper contain erroneous wave forms and cannot be relied upon to provide a check of simulations (e.g., see Boore and Zoback, 1974, Figure 6).

DEFINITIONS

$$r_2 = x_2 - \xi_2 \tag{2a}$$

$$r_3 = x_3 \tag{2b}$$

$$a^2 = r_2{}^2 + r_3{}^2 \tag{2c}$$

$$r_m = \frac{c^2(x_1-vt)+v(a^2(c^2-v^2)+c^2(x_1-vt)^2)^{1/2}}{(c^2-v^2)} \tag{3}$$

$$t_{Om} = \frac{(x_1{}^2+a^2)^{1/2}}{c} \quad (4a) \qquad t_{Lm} = \frac{[(x_1-L)^2+a^2]^{1/2}}{c}+\frac{L}{v} \quad (4b)$$

$$\begin{aligned} \operatorname{sgn}(m) &= +1 \text{ for } m = p \\ &-1 \text{ for } m = s. \end{aligned} \tag{5}$$

In equations (3) to (5) above, m designates the wave mode, either P or S, whose corresponding velocity is c, either α or β. r_m is a function of time and corresponds to the root r_1 of the equation,

$$t = \frac{x_1-r_1}{v}+\frac{(r_1{}^2+a^2)^{1/2}}{c}.$$

This equation gives the time required for rupture along the fault from $\xi_1 = 0$ to $\xi_1 = x_1-r_1$, plus the time to travel to the observation point as either a P- or a S-wave. Thus when the equation is inverted, the physical meaning of r_m is that at a given time t, x_1-r_m represents the distance along the rupture front from which a radiated wave of mode m will arrive at the observation point at the given time t. t_{Om} and t_{Lm} correspond to the time of the arrival of the m wave from the ends of the strip at $x_1 = 0$ and $x_1 = L$, respectively.

$$H_{ik}(y_1, y_2) = H_{ik}(y_1)-H_{ik}(y_2) \tag{6}$$

$$F^m_{ik}(y_1, y_2) = F^m_{ik}(y_1)-F^m_{ik}(y_2) \tag{7}$$

where y_1 and y_2 are dummy variables, corresponding to ζ in the listing of the functions in equations (12) to (17).

SOLUTION

To evaluate the total wave motion, we simply sum the wave motions for each mode,

$$u_i = u_i^p+u_i^s \tag{8}$$

where the displacements due to each mode are given by (for the ranges listed):

For the range $t < t_{Om}$ (or $x_1 < r_m$),

$$u_i^m = 0 \tag{9a}$$

For the range $t_{Om} < t < t_{Lm}$ (or $x_1-L < r_m < x_1$),

$$u_i^m = \frac{\beta^2}{4\pi}\sum_{k=1}^{2} D_k \{I^m_{ik}(r_m)+\operatorname{sgn}(m)H_{ik}(x_1, r_m)+F^m_{ik}(x_1, r_m)\} \tag{9b}$$

For the range $t_{Lm} < t$ (or $r_m < x_1-L$),

$$u_i^m = \frac{\beta^2}{4\pi}\sum_{k=1}^{2} D_k \{\operatorname{sgn}(m)H_{ik}(x_1, x_1-L)+F^m_{ik}(x_1, x_1-L)\}. \tag{9c}$$

Using the dummy variable ζ and the ζ dependent definitions,

$$r^2 = \zeta^2 + a^2 \tag{10}$$

$$\theta(c) = \frac{vc}{c - v\zeta/r} \tag{11}$$

The functions $I_{ik}^m(\zeta)$, $F_{ik}^m(\zeta)$ and $H_{ik}(\zeta)$ are given below.

The $I_{ik}^m(\zeta)$ terms for

Strike-slip *Dip-slip*

P mode

$$I_{11}^p(\zeta) = \frac{2\zeta^2 r_3}{\alpha^3 r^4}\,\theta(\alpha) \qquad I_{12}^p(\zeta) = \frac{2\zeta r_2 r_3}{\alpha^3 r^4}\,\theta(\alpha)$$

$$I_{21}^p(\zeta) = \frac{2\zeta r_2 r_3}{\alpha^3 r^4}\,\theta(\alpha) \qquad I_{22}^p(\zeta) = \frac{2 r_2^2 r_3}{\alpha^3 r^4}\,\theta(\alpha)$$

$$I_{31}^p(\zeta) = \frac{2\zeta r_3^2}{\alpha^3 r^4}\,\theta(\alpha) \qquad I_{32}^p(\zeta) = \frac{2 r_2 r_3^2}{\alpha^3 r^4}\,\theta(\alpha) \tag{12}$$

S mode

$$I_{11}^s(\zeta) = \frac{r_3}{\beta^3 r^2}\left(1 - \frac{2\zeta^2}{r^2}\right)\theta(\beta) \qquad I_{12}^s(\zeta) = \frac{-2\zeta r_2 r_3}{\beta^3 r^4}\,\theta(\beta)$$

$$I_{21}^s(\zeta) = \frac{-2\zeta r_2 r_3}{\beta^3 r^4}\,\theta(\beta) \qquad I_{22}^s(\zeta) = \frac{r_3}{\beta^3 r^2}\left(1 - \frac{2 r_2^2}{r^2}\right)\theta(\beta)$$

$$I_{31}^s(\zeta) = \frac{\zeta}{\beta^3 r^2}\left(1 - \frac{2 r_3^2}{r^2}\right)\theta(\beta) \qquad I_{32}^s(\zeta) = \frac{r_2}{\beta^3 r^2}\left(1 - \frac{2 r_3^2}{r^2}\right)\theta(\beta) \tag{13}$$

The $F_{ik}^m(\zeta)$ terms,

Strike-slip *Dip-slip*

P mode

$$F_{11}^p(\zeta) = \frac{\zeta r_3}{\alpha^2 a^2 r}\left(1 - \frac{\zeta^2}{r^2}\right) \qquad F_{12}^p(\zeta) = \frac{r_2 r_3}{\alpha^2 r^3}$$

$$F_{21}^p(\zeta) = \frac{r_2 r_3}{\alpha^2 r^3} \qquad F_{22}^p(\zeta) = \frac{\zeta r_3}{\alpha^2 a^2 r}\left\{1 + \frac{r_2^2}{a^2}\left(\frac{\zeta^2}{r^2} - 3\right)\right\}$$

$$F_{31}^p(\zeta) = \frac{1}{\alpha^2 r}\left(\frac{r_3^2}{r^2} - 1\right) \qquad F_{32}^p(\zeta) = \frac{\zeta r_2}{\alpha^2 a^2 r}\left\{1 + \frac{r_3^2}{a^2}\left(\frac{\zeta^2}{r^2} - 3\right)\right\} \tag{14}$$

S mode *Strike-slip* *Dip-slip*

$$F^s_{11}(\zeta) = \frac{\zeta^3 r_3}{\beta^2 a^2 r^3} \qquad F^s_{12}(\zeta) = -\frac{r_2 r_3}{\beta^2 r^3}$$

$$F^s_{21}(\zeta) = -\frac{r_2 r_3}{\beta^2 r^3} \qquad F^s_{22}(\zeta) = \frac{\zeta r_2^2 r_3}{\beta^2 a^4 r}\left(3-\frac{\zeta^2}{r^2}\right)$$

$$F^s_{31}(\zeta) = -\frac{r_3^2}{\beta^2 r^3} \qquad F^s_{32}(\zeta) = \frac{\zeta r_2 r_3^2}{\beta^2 a^4 r}\left(3-\frac{\zeta^2}{r^2}\right) \tag{15}$$

Finally, the H_{ik} terms are given below. (The notation should not be confused with the Heaviside step notation of equation 1.)

Strike-slip

$$H_{11}(\zeta) = r_3\left\{\frac{1}{v^2a^2}\frac{\zeta^3}{r^3}\left(\frac{3\zeta^2}{r^2}-1\right)+\frac{2(t-x_1/v)}{vr^3}\left(\frac{3a^2}{r^2}-4\right)\right.$$
$$\left.-\frac{3(t-x_1/v)^2}{a^4}\frac{\zeta}{r}\left(\frac{\zeta^4}{r^4}-\frac{2\zeta^2}{r^2}+1\right)\right\}$$

$$H_{21}(\zeta) = r_2 r_3\left\{\frac{1}{v^2r^3}\left(\frac{3a^2}{r^2}-5\right)+\frac{2(t-x_1/v)}{va^4}\frac{\zeta^3}{r^3}\left(5-\frac{3\zeta^2}{r^2}\right)\right.$$
$$\left.-\frac{3}{r^5}(t-x_1/v)^2\right\}$$

$$H_{31}(\zeta) = \frac{r_3^2}{v^2r^3}\left(\frac{3a^2}{r^2}-5\right)+\frac{1}{v^2r}\left(3-\frac{a^2}{r^2}\right)-\frac{2(t-x_1/v)}{va^2}\frac{\zeta^3}{r^3}$$
$$+\frac{2r_3^2}{va^4}(t-x_1/v)\frac{\zeta^3}{r^3}\left(5-\frac{3\zeta^2}{r^2}\right)+\frac{(t-x_1/v)^2}{r^3}\left(1-\frac{3r_3^2}{r^2}\right) \tag{16a}$$

Dip-slip

$$H_{12}(\zeta) = H_{21}(\zeta)$$

$$H_{22}(\zeta) = r_3\left\{\frac{r_2^2\zeta^3}{v^2a^4r^3}\left(5-\frac{3\zeta^2}{r^2}\right)-\frac{\zeta^3}{v^2a^2r^3}+\frac{2(t-x_1/v)}{vr^3}\left(1-\frac{3r_2^2}{r^2}\right)\right.$$
$$\left.+\frac{r_2^2}{a^6}(t-x_1/v)^2\frac{\zeta}{r}\left(15-\frac{10\zeta^2}{r^2}+\frac{3\zeta^4}{r^4}\right)+\frac{(t-x_1/v)^2}{a^4}\frac{\zeta}{r}\left(\frac{\zeta^2}{r^2}-3\right)\right\}$$

$$H_{32}(\zeta) = r_2\left\{\frac{r_3^2\zeta^3}{v^2a^4r^3}\left(5-\frac{3\zeta^2}{r^2}\right)-\frac{\zeta^3}{v^2a^2r^3}+\frac{2(t-x_1/v)}{vr^3}\left(1-\frac{3r_3^2}{r^2}\right)\right.$$
$$+\frac{r_3^2}{a^6}(t-x_1/v)^2\frac{\zeta}{r}\left(15-\frac{10\zeta^2}{r^2}+\frac{3\zeta^4}{r^4}\right)+\frac{(t-x_1/v)^2}{a^4}\frac{\zeta}{r}$$
$$\left.\times\left(\frac{\zeta^2}{r^2}-3\right)\right\}. \tag{16b}$$

The computation of a seismogram for a strip at ξ_2, at particular time (t) and observation point (x_1, x_2, x_3), begins with the evaluation of of r_α and r_β from equation (3). These values are then used as arguments in equations (9), where the individual terms are defined in equations (12), (14), and (16) (for the P mode) and in equations (13), (15), and (16) (for the S mode). The contributions for each mode are then summed according to equation (8). The result has dimensions of (units of dislocation)/(units of fault width). After the time series for each strip is evaluated, the total response is obtained by summing the contributions of each strip. This summation corresponds to a numerical integration and as such can be done with varying complexity. The final step is a convolution with the time derivative of the desired source time-function.

The step-function response contains step discontinuities at the arrival of discrete phases from the beginning and end of the fault, and in some cases it may be desirable to evaluate the step-function response for each strip at unequal time steps such that the discontinuities fall on the discrete time points. In this way the discontinuities can be represented accurately with relatively large time spacing. Since each strip will contain discontinuities at different times, it may be most convenient to smooth, by convolution, the time series corresponding to each strip and interpolate to a common time base before summing over all the strips. This degree of sophistication is usually not necessary unless several numerical derivatives of the motion are taken (as in constructing an accelerogram), in which case the high-frequency components are accentuated. When working with displacement simulations, the number of strips required to adequately represent the motion produced by the finite width of the fault can be very small (1 to 3).

Acknowledgment

This research was supported by the Earth Sciences Section, National Science Foundation, NSF Grant GA 36135XI.

References

Anderson, J. (1974). A dislocation model for the Parkfield earthquake, *Bull. Seism. Soc. Am.* **64,** 671–686.

Battis, J. C. and L. S. Turnbull, Jr. (1974). Analysis of two California earthquakes using a moving dislocation source, *EOS (abstract)*, **55,** 352.

Boore, D. and M. D. Zoback (1974). Near-field motions from kinematic models of propagating faults, *Bull. Seism. Soc. Am.* **64,** 321–342.

Haskell, N. A. (1969). Elastic displacements in the near-field of a propagating fault, *Bull. Seism. Soc. Am.* **59,** 865–908.

Knopoff, L. and F. Gilbert (1959). Radiation from a strike-slip fault, *Bull. Seism. Soc. Am.* **49,** 163–178.

Savage, J. C. (1965). The effect of rupture velocity on seismic first motions, *Bull. Seism. Soc. Am.* **55,** 263–275.

Trifunac, M. D. (1974). A three-dimensional dislocation model for the San Fernando, California, earthquake of February 9, 1971, *Bull. Seism. Soc. Am.* **64,** 149–172.

Tsai, Y. B. and H. J. Patton (1972). Near field small earthquakes—dislocation motion, *Semi-Annual Report 1*, Texas Instr. Inc. (prep. for Air Force Off. Sci. Res., Contract F44620–72–C–0073).

Department of Geophysics
Stanford University
Stanford, California 94305

Manuscript received June 3, 1974